Evaluation of Measurement Uncertainty

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Agenda

- Basic Concepts
- Uncertainty Evaluation – the past
- Uncertainty Evaluation – the present
- Uncertainty Evaluation – the future

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Basic Concepts

Measurement (VIM 3)

- Process of experimentally obtaining one or more quantity values that can reasonably be attributed to a quantity

Basic Concepts

Calibration (VIM3)

- operation that, under specified conditions, in a first step, establishes a relation between the quantity values with measurement uncertainties provided by measurement standards and corresponding indications with associated measurement uncertainties and, in a second step, uses this information to establish a relation for obtaining a measurement result from an indication

Basic Concepts

- Measurement and calibration are not meaningful without specifying the measurement uncertainties

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- Uncertainty Evaluation – the past
- Uncertainty Evaluation – the present
- Uncertainty Evaluation – the future

Uncertainty Evaluation – the Past

- Error analysis
- True value
- Bias (systematic error)
- Dispersion (random error)

Uncertainty Evaluation – the Past

- True value
- Errors (Systematic and Random)
- Ideal concepts (unknown and unknowable)
- Recommendation INC-1 (1980) to address this problem
- Publication of the Guide to the Expression of Uncertainty in Measurement (GUM)

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- Basic Concepts
- Uncertainty Evaluation – the past
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Uncertainty Evaluation – the present

- GUM (1993)
- Based on observable concepts
- Uncertainty – a parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand.

Brief Description of GUM

- Define measurand
- Define measurement model
- Type A evaluation
- Type B evaluation
- Combined standard uncertainty
- Find coverage factor and calculate the expanded uncertainty

Introduction to GUM
(Evaluation of Uncertainties)

The Mathematical Model

Express the output variable y in terms of input variables x_1, x_2, … x_n, such that:

$Y = f(X_1, X_2, X_3, \ldots, X_n)$

e.g. modelling for current measurement

$I = V/R$

What are the influencing factors?

Introduction to GUM
(Evaluation of Uncertainties)

Uncertainty for each input variable

- Type A evaluation
  – by statistical means
- Type B evaluation
  – by other means

Introduction to GUM
(Evaluation of Uncertainties)

Evaluation of Type A Uncertainty

- Calculate the standard deviation s and the experimental standard deviation of the mean (ESDM)

$$s^2(q) = \frac{1}{n-1}\sum_{k=1}^{n}(q_k - \bar{q})^2$$

$$u(q) = ESDM = \frac{s\ (q)}{\sqrt{n}}$$

Introduction to GUM
(Evaluation of Uncertainties)

Evaluation of Type B Uncertainty

- Based on known priori
- With rectangular distribution and semi-range equal to a

$$u(q) = \frac{a}{\sqrt{3}}$$

Introduction to GUM
(Evaluation of Uncertainties)

Evaluation of Type B Uncertainty

- Based on previous calibration certificate
- With expanded uncertainty a with a coverage factor of 2

$$u(q) = \frac{a}{2}$$

Introduction to GUM
(Combined Standard Uncertainty, u_c)

Uncorrelated input quantities

$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i)$$

Correlated input quantities

$$u_c^2(y) = \sum_{j=1}^{N} \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right) \left(\frac{\partial f}{\partial x_j}\right) \mathrm{cov}\ (x_i, x_j)$$

Introduction to GUM
(Expanded Uncertainty, U)

- Expanded uncertainty U for a level of confidence of 95 % is given by :

$$U = k \times u_c(y)$$

where k is the coverage factor

Limitations of GUM Framework

Assumptions

1. Model is linear
2. Input quantities are symmetrical
3. No dominant contributions
4. Output quantity is symmetrical and Gaussian or t-distributed

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Uncertainty Evaluation – the Future

- BIPM establishes a Joint Committee for Guides in Metrology (JCGM) to address concerns for GUM and VIM
- Working Group 1 (WG1) of the Joint Committee for Guides in Metrology (JCGM) is responsible for GUM

Uncertainty Evaluation – the Future

- JCGM-WG1 has decided to produce a series of complementary documents and supplements to cover some topics of interest in more detail.

GUM Documentation Structure

- Fundamental document
- GUM Supplements
- Complementary Documents

Fundamental Document – The GUM

- First edition published in 1993
- 1st edition, 2nd print (1995)
- 1st edition, 3rd print (2008), as JCGM100:2008

GUM Supplements

- Extensions to the GUM, to be used in conjunction with JCGM100
- Supplements foreseen/published
  - JCGM 101: Supplement 1 to the GUM – Propagation of distributions using a Monte Carlo method (2008);
  - JCGM 102: Supplement 2 to the GUM – Models with any number of output quantities (2011),
  - JCGM 103: Supplement 3 to the GUM – Developing and using measurement models

GUM Supplement 1

- Propagation of pdfs by Monte Carlo Simulation

GUM Supplement 1

- Versatile propagation method, capable of dealing with
  - Non-linear models
  - Models with constraints
- From output pdf, desired output can be calculated, e.g.,
  - Coverage interval
  - Standard uncertainty

GUM Supplement 2

- Extension to models with any number of output quantities
  - Uncertainty propagation (GUF)
  - Propagation of pdfs (GUM-S1)
- Use of complex numbers
- Determination of confidence region
- Examples: electrical, mechanical and acoustic measurements with magnitude and phase

GUM Supplement 3

- Describes measurement modelling and use of models
- Document in an early stage of development
- 'Fishbone' diagrams as modeling aid to be included ('cause and effect' modelling)

Complementary Documents

- Giving background, introduction, further guidance to aspects dealt with in the GUM
- Documents foreseen/published
  - JCGM 104: Evaluation of measurement data – An introduction to the "Guide to the expression of uncertainty in measurement" and related documents (2009)
  - JCGM 105: Evaluation of measurement data – Concepts and basic principles
  - JCGM 106: Evaluation of measurement data – The role of measurement uncertainty in conformity assessment,
  - JCGM 107: Evaluation of measurement data – Applications of the least-squares method

JCGM 104: 2009

- Introduction to the GUM
- Explanatory document
  - Concepts and principles
  - Stages of uncertainty evaluation
  - Formulation stage
  - Propagation of uncertainty
  - Conformity assessment
  - Least squares

JCGM 105

- Concepts, principles underlying the GUM
- Document in an early stage of development
- Support to choices made in GUM and its supplements

JCGM 106

- Use of measurement uncertainty in conformity assessment
- Methodologies for decision taking on the basis of results including uncertainty
- Draft of the document available

JCGM 107

- Least square method
- Calibration curve
- Calibration parameters

Revision of the GUM

To address the following problems:

- Inconsistencies
  - Internally (frequentist and Bayesian approaches; terminology)
  - Externally (GUM Supplements; VIM3)
- Inadequacies
  - Measurement uncertainty evaluation in new fields
  - Concept of a unique true value
- Ambiguities
  - Notational and terminology

Objectives of the revision

- Clarity of presentation
- Structure as close as possible to that of the present GUM
- Level of presentation comparable to that of the present GUM
- Better specification of the conditions of applicability

Outcome of revision

- Unification of the concepts of Type A and Type B evaluations of uncertainty
- Increased guidance in the evaluation of standard uncertainty for input estimates
- Links to GUM Supplements
- Increased number of examples, with applications taken from chemistry etc

Available by the end of 2014

Thank You