

The Role of Measurement Uncertainty in  
Conformity Assessment  
測量不確定度在合格評定的角色

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## Agenda

- Introduction
- Simple yet naïve approach
- Intuitive approach
- Worst case (maximum risk) approach
- Global (expected) risk approach

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## Simple Question

- Does the measurement result meet the specification?

Yet it is difficult to answer !

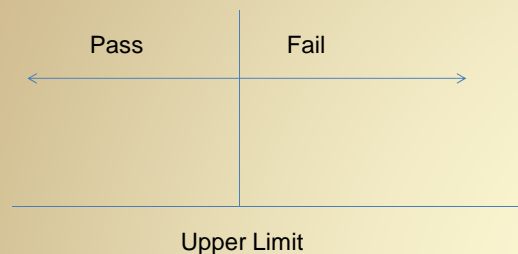
Because every measurement has uncertainty associated with it.

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## Simple yet naïve approach

- No consideration of measurement uncertainty
- Result  $>$  limit  $\Rightarrow$  Fail
- Result  $\leq$  limit  $\Rightarrow$  Pass

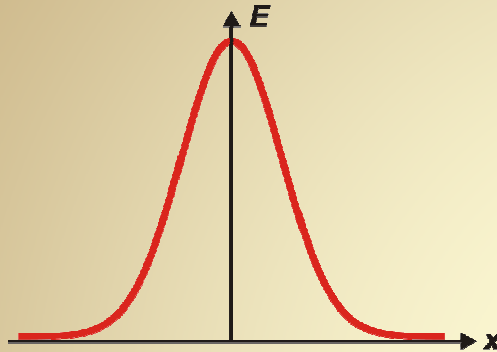


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## Problem with this approach

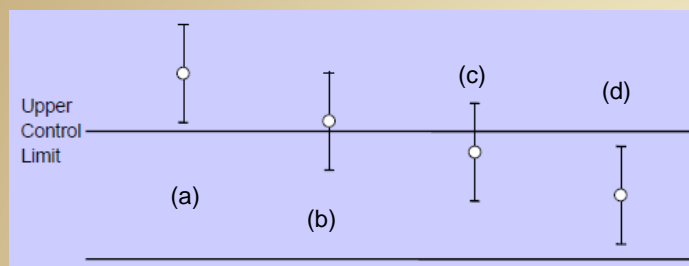
- Risk of wrong decision is 50 % at specification limit.



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## Intuitive Approach



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## Intuitive approach

- (a) Out of limit
- (b) Unable to judge
- (c) Unable to judge
- (d) Within limit

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## Intuitive approach

- ISO 14253-1
- ILAC-G8

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## Effect of uncertainty

- The greater the measurement uncertainty (MU), the larger the area of unclear decision
- Test uncertainty ratio (TUR)

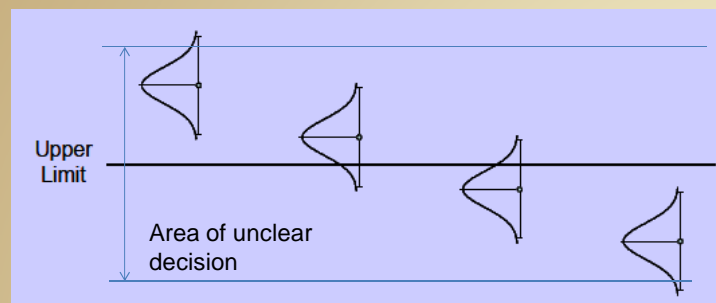
$$\text{TUR} = \text{Spec Limit} / \text{MU}$$

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## Effect of MU



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## Maximum risk approach

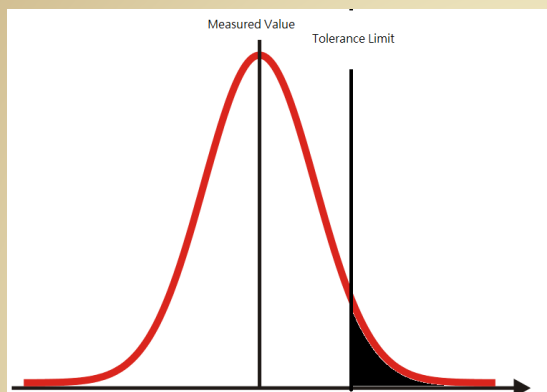
- Consumer's risk – risk of accepting non-conforming items (product, process, system, person etc)
- Producer's risk – risk of rejecting conforming items

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## Specific consumer's risk

- Consumer's risk at a particular measured value



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## Specific Consumer risk

$$R_C^* = 1 - \Theta(y_m)$$

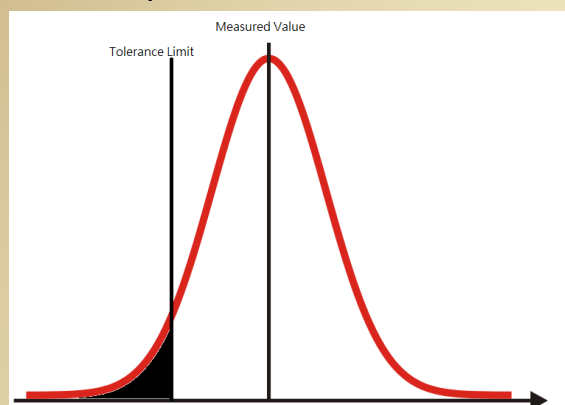
- $\Theta$  is the cumulative distributive function
- $y_m$  is the measured value

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## Specific Producer's risk

- Producer's risk at a particular measured value

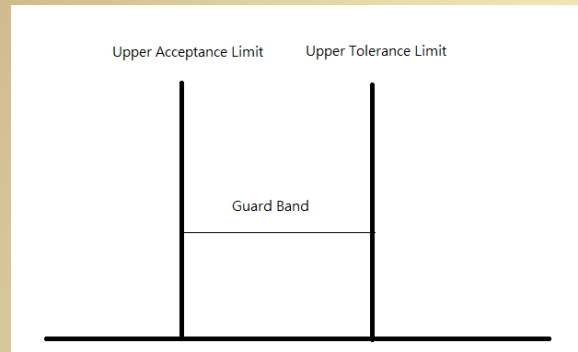


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## Guarded Acceptance

- Upper acceptance limit ( $A_u$ ) < Upper tolerance limit ( $T_u$ ) to reduce the consumer's risk

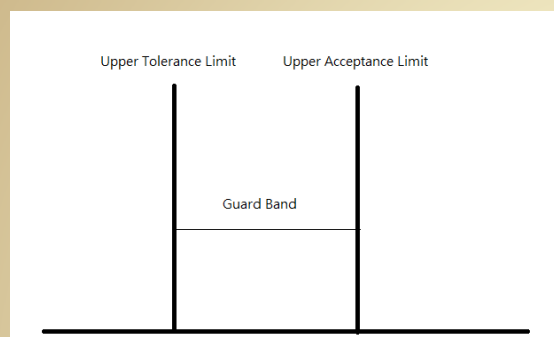


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## Guarded Rejection

- Upper acceptance limit > upper tolerance limit so reduce the producer's risk



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## Guard Band Calculation

- Worst case scenario (maximum risk)
- Specific consumer's risk at acceptance limit
- Guard band width  $w = ku$
- $RC^* = 95\%$  when  $k = 1.65$  (normal distribution)
- $RC^* = 99\%$  when  $k = 2.33$  (normal distribution)

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## Problems of this approach

- Worst case scenario
- Very conservative since not all measured value falls near the limits

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## Global risk approach

- Consider the distribution of measured value
- Expected (average) risk
- Based on Bayesian statistics
- JCGM 106

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## JCGM

- The Joint Committee for Guides in Metrology is a committee set up in 1997 to address common problems encountered in metrology. It was created by seven international organizations responsible for two important documents, the Guide to the Expression of uncertainty in measurement (GUM) and the International vocabulary of basic and general terms in metrology (VIM).

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## JCGM

- There are two working groups under JCGM. Working Group 1 is responsible for promoting the use of the GUM and preparing Supplements and other documents for its broad application. Working Group 2 is tasked to revise and promote the use of the VIM.

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## JCGM Documents

- JCGM 100 – Evaluation of measurement data – Guide to the expression of uncertainty in measurement (GUM).
- JCGM 101 – Evaluation of measurement data – Supplement 1 to the “Guide to the expression of uncertainty in measurement” - Propagation of distribution using a Monte Carlo method.

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## JCGM Documents

- JCGM 102 - Evaluation of measurement data – Supplement 1 to the “Guide to the expression of uncertainty in measurement” - Extension to any number of output quantities.
- JCGM 104 - Evaluation of measurement data – An introduction to the “Guide to the expression of uncertainty in measurement” and its supplements.

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## JCGM Documents

- JCGM 106 - Evaluation of measurement data – The role of measurement uncertainty in conformity assessment.
- JCGM 200 – International vocabulary of metrology – Basic and general concepts and associated terms (VIM).

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## Bayesian Statistics

- Probability is the degree of believe
- Prior knowledge
- Additional information gained (likelihood function) through measurement
- Updated knowledge (posterior)

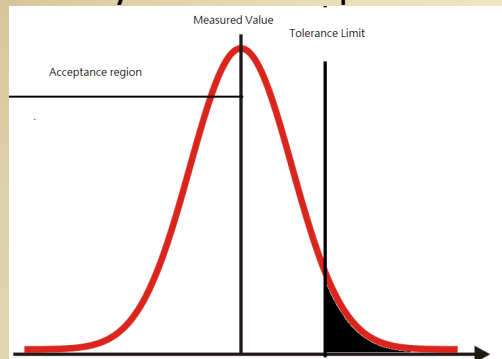
$$g(y/y_m) = C g_0(y) h(y_m|y)$$

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## Global consumer's risk

- Probability of accepting a non-conforming item chosen randomly within acceptance limits



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## Global consumer's risk

$$R_C = \int_{\tilde{C}} \int_A g_0(y) h(y_m|y) dy_m dy$$

$$R_C = \int_A R_C^*(y_m) g(y_m) dy_m$$

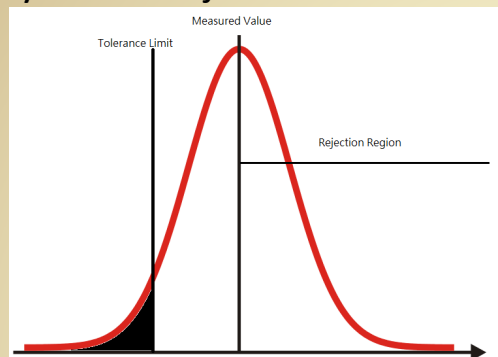
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## Global Producer's Risk

- Probability of rejecting a conforming item chosen randomly within rejection limits



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## Global Producer's Risk

$$R_P = \int_C \int_{\tilde{A}} g_0(y) h(y_m | y) dy_m dy$$

$$R_P = \int_{\tilde{A}} R_P^*(y_m) g(y_m) dy_m$$

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## Limitations of the JCGM 106

- Supports only a specific distribution type, the Gaussian like distribution.
- Only graphical solution provided to find the acceptance limits for specific distribution parameters

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## SCL's Paper to be presented at NCSLI 2013

### Software for Implementation of JCGM 106

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#### Abstract

A new guidance document "Evaluation of measurement data – The role of measurement uncertainty in conformity assessment" prepared by the Joint Committee for Guides in Metrology was published in October 2012. The document provides guidance and procedures for determining an acceptance interval, chosen so as to balance the risks associated the consumers and the producers. The Standards and Calibration Laboratory (SCL), Hong Kong has developed a software tool that allows easy calculation of the acceptance limits based on the production process, the measurement system capabilities, and the defined consumer or producer risks.

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## Improvements made by SCL

- Supports many commonly used distribution types (rectangular, normal, t, triangular, log-normal, gamma, etc)
- Numerical solution provided to find the acceptance limits

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## Description of the algorithms

- RCS – calculate the specific consumer's risk
- RPS – calculate the specific producer's risk
- GRC – calculate the global consumer's risk
- GRP – calculate the global producer's risk
- RGRC – calculate the acceptance limits based on specified global consumer's risk
- RGRP – calculate the acceptance limits based on specified global producer's risk

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Thank You

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